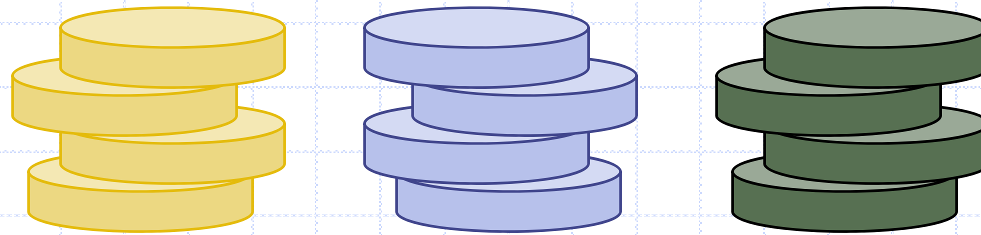
Stacks



Outline and Reading

- The Stack ADT (4.2.1)
- Applications of Stacks (4.2.3)
- Array-based implementation (4.2.2)
- Growable array-based stack

Abstract Data Types (ADTs)

- An abstract data type (ADT) is an abstraction of a data structure
- An ADT specifies:
  - Data stored
  - Operations on the data
  - Error conditions associated with operations
- Example: ADT modeling a simple stock trading system
  - The data stored are buy/sell orders
  - The operations supported are
    - order buy(stock, shares, price)
    - order sell(stock, shares, price)
    - void cancel(order)
  - Error conditions:
    - Buy/sell a nonexistent stock
    - Cancel a nonexistent order

The Stack ADT

- The Stack ADT stores arbitrary objects
- Insertions and deletions follow the last-in first-out scheme
- Think of a spring-loaded plate dispenser
- Main stack operations:
  - push(object o): inserts element o
  - pop(): removes and returns the last inserted element
- Auxiliary stack operations:
  - top(): returns a reference to the last inserted element without removing it
  - size(): returns the number of elements stored
  - isEmpty(): returns a Boolean value indicating whether no elements are stored

Exceptions

- Attempting the execution of an operation of ADT may sometimes cause an error condition, called an exception
- Exceptions are said to be "thrown" by an operation that cannot be executed
- In the Stack ADT, operations pop and top cannot be performed if the stack is empty
- Attempting the execution of pop or top on an empty stack throws an EmptyStackException

Applications of Stacks

- Direct applications
  - Page-visited history in a Web browser
  - Undo sequence in a text editor
  - Saving local variables when one function calls another, and this one calls another, and so on.
- Indirect applications
  - Auxiliary data structure for algorithms
  - Component of other data structures

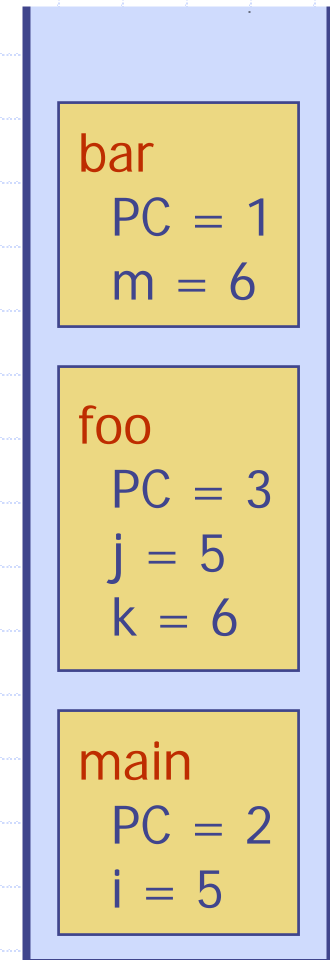
C++ Run-time Stack

- The C++ run-time system keeps track of the chain of active functions with a stack
- When a function is called, the run-time system pushes on the stack a frame containing
  - Local variables and return value
  - Program counter, keeping track of the statement being executed
- When a function returns, its frame is popped from the stack and control is passed to the method on top of the stack

```
main() {
  int i = 5;
  foo(i);
}

foo(int j) {
  int k;
  k = j+1;
  bar(k);
}

bar(int m) {
  …
}
```

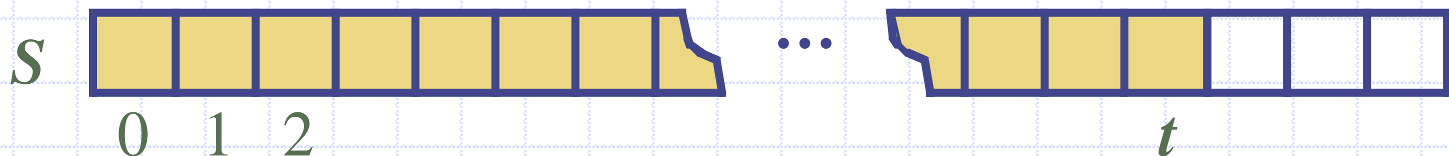


Array-based Stack

- A simple way of implementing the Stack ADT uses an array
- We add elements from left to right
- A variable keeps track of the index of the top element

Algorithm *size*()
  return $t + 1$

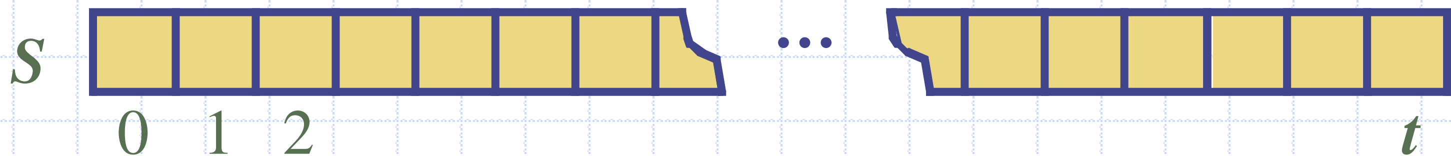
Algorithm *pop*()
  if *isEmpty*() **then**
    throw *EmptyStackException*
  else
    $t \leftarrow t - 1$
    return $S[t + 1]$



Array-based Stack (cont.)

- The array storing the stack elements may become full
- A push operation will then throw a FullStackException
  - Limitation of the array-based implementation
  - Not intrinsic to the Stack ADT

Algorithm *push*(o)
  if $t = S.length - 1$ **then**
    throw *FullStackException*
  else
    $t \leftarrow t + 1$
    $S[t] \leftarrow o$



Performance and Limitations

- Performance
  - Let n be the number of elements in the stack
  - The space used is $O(n)$
  - Each operation runs in time $O(1)$
- Limitations
  - The maximum size of the stack must be defined *a priori* , and cannot be changed
  - Trying to push a new element into a full stack causes an implementation-specific exception

Growable Array-based Stack

- In a push operation, when the array is full, instead of throwing an exception, we can replace the array with a larger one
- How large should the new array be?
  - incremental strategy: increase the size by a constant c
  - doubling strategy: double the size

Algorithm *push*(*o*)
  if $t = S.length - 1$ **then**
    $A \leftarrow$ new array of size …
    for $i \leftarrow 0$ **to** t **do**
      $A[i] \leftarrow S[i]$
    $S \leftarrow A$
  $t \leftarrow t + 1$
  $S[t] \leftarrow o$

Stack Interface in C++

- Interface corresponding to our Stack ADT
- Requires the definition of class EmptyStackException
- Most similar STL construct is vector

```
template <typename Object>
class Stack {
public:
    int size();
    bool isEmpty();
    Object& top()
        throw(EmptyStackException);
    void push(Object o);
    Object pop()
        throw(EmptyStackException);
};
```

Array-based Stack in C++

```
template <typename Object>
class ArrayStack {
private:
    int capacity;       // stack capacity
    Object *S;          // stack array
    int top;            // top of stack
public:
    ArrayStack(int c) {
        capacity = c;
        S = new Object[capacity];
        t = –1;
    }
```

```
    bool isEmpty()
      { return (t < 0); }

    Object pop()
          throw(EmptyStackException) {
        if(isEmpty())
          throw EmptyStackException
            ("Access to empty stack");
          return S[t--];
    }
// … (other functions omitted)
```